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Are Nonreciprocal Bi-Isotropic Media Forbidden Indeed?

Ari H. Sihvola

***Abstract*— Doubt is cast in this article on the universality of the conclusion that linear bi-isotropic media have to be reciprocal, which claim has recently been set forth by Lakhtakia and Weiglhofer.**

I. Introduction

Bianisotropic materials have a more complicated response to electricity than ordinary isotropic materials. The polarization behavior of bianisotropic media is contained in the constitutive relations between the electric ($\bar{D}$) and magnetic ($\bar{B}$) displacements, and the electric ($\bar{E}$) and magnetic ($\bar{H}$) field vectors

$$\bar{D} = \bar{\bar{\epsilon}} \cdot \bar{E} + \bar{\bar{\xi}} \cdot \bar{H} \tag{1}$$
$$\bar{B} = \bar{\bar{\zeta}} \cdot \bar{E} + \bar{\bar{\mu}} \cdot \bar{H}. \tag{2}$$

Here the material parameter dyadics are permittivity $\bar{\bar{\epsilon}}$, permeability $\bar{\bar{\mu}}$, and the magnetoelectric crosspolarisations $\bar{\bar{\xi}}$ and $\bar{\bar{\zeta}}$. Nonisotropy makes the four material parameter relations dyadic with nine components in general, and the total number of independent material parameters of bianisotropic media is hence 36.

The material parameters are limited by principles dictated by physical laws. For example, the well-known causality requirement leads to the Kronig–Kramers equations. And since the results of bianisotropic electromagnetics are relevant to the growing number of microwave applications of complex materials, it is evermore important to find out and articulate physical restrictions for the magnetoelectric parameters.

A recent publication [1] provides us with an argument that gives one restriction to the 36 parameters for linear bianisotropic media. Lakhtakia and Weiglhofer use tensor analysis and the general covariance requirements given by Post [2] for uniform media and structural fields, and the claim resulting from their analysis is strong: the sum of the traces of the magnetoelectric dyadics must vanish. Applied to the special case of bi-isotropic media[1], the conclusion of the analysis in [1] is that isotropic materials have to be reciprocal. In other words, the claim stands that NRBI (nonreciprocal bi-isotropic materials) cannot exist.

It is the purpose of this article to discuss possible counterexamples to this NRBI-nonexistence result. For that end, let us reformulate the constitutive relations (1)–(2) into a form where reciprocity becomes visible

$$\bar{D} = \bar{\bar{\epsilon}} \cdot \bar{E} + (\bar{\bar{\chi}}^T - j\bar{\bar{\kappa}}^T)\sqrt{\mu_0\epsilon_0} \cdot \bar{H} \tag{3}$$
$$\bar{B} = (\bar{\bar{\chi}} + j\bar{\bar{\kappa}})\sqrt{\mu_0\epsilon_0} \cdot \bar{E} + \bar{\bar{\mu}} \cdot \bar{H} \tag{4}$$

where now the magnetoelectric parameters are contained in the chirality dyadic $\bar{\bar{\kappa}}$ and the nonreciprocity dyadic $\bar{\bar{\chi}}$. The superscript T denotes the transpose operation.[2] The nonreciprocity decomposition (3)–(4) is in accord with the reciprocity definition for bianisotropic media [4]

$$\bar{\bar{\epsilon}} = \bar{\bar{\epsilon}}^T, \quad \bar{\bar{\mu}} = \bar{\bar{\mu}}^T, \quad \bar{\bar{\xi}} = -\bar{\bar{\zeta}}^T, \quad \text{(for reciprocal media).} \tag{5}$$

Bi-isotropic media have material dyadics that are multiples of a unit dyadic $\bar{\bar{I}}$. The well-known isotropic chiral medium has dyadics $\bar{\bar{\kappa}} = \kappa\bar{\bar{I}}$ and $\bar{\bar{\chi}} = 0$ [3]. And in particular, the nonreciprocity dyadic of a sample of NRBI material is of the form $\bar{\bar{\chi}} = \chi\bar{\bar{I}}$, where $\chi \neq 0$.

II. Consequences of the Magnetoelectric Tracelessness

The crucial result of [1] is a condition for the trace of the magnetoelectric dyadics. In particular, it restricts the nonreciprocal part of these dyadics with the condition

$$\mathrm{tr}\{\bar{\bar{\chi}}\} = 0 \tag{6}$$

where trace means the sum of the diagonal elements of the dyadic. Note that the constitutive relations used here relate the pair $(\bar{D}, \bar{B})$ to the pair $(\bar{E}, \bar{H})$ whereas [1] follows the "Boys–Post" relations where $\bar{D}$ and $\bar{H}$ are given as material functions of the primary fields $\bar{E}$ and $\bar{B}$. However, the magnetoelectric parameters have the same

[1] For bi-isotropic media, the four dyadics reduce to scalars since there is no direction dependence in the medium. The magnetoelectric coupling remains through two parameters.

[2] In (3)–(4), the additional coefficients (the imaginary unit j and the free-space parameters μ_0, ϵ_0) have been included for conformity with earlier notation [3].

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meaning for nonmagnetic materials ($\bar{\bar{\mu}} = \mu_0 \bar{\bar{I}}$, which is the case in the following analysis) in both systems.[3]

It is clear that the tracelessness condition prohibits the existence of NRBI since $\mathrm{tr}\{\bar{\bar{\chi}}\} = 0$ leads to $\chi = 0$ in case of bi-isotropy. As a matter of fact, the condition can even be used as an effective refutation of the following argument that has been presented to construct a sample of NRBI medium.

Take a piece of nonreciprocal bianisotropic material (e.g., chromium oxide), and cut it into pieces. Then mix these inclusions, randomly oriented, in isotropic matrix material, like epoxy resin. Hence the anisotropic nonreciprocity of the original inclusions will be averaged within the mixture that certainly becomes isotropic.

However, the effective nonreciprocity of the final mixture is proportional to one-third of the trace of the inclusion nonreciprocity dyadic. Therefore, if the trace vanishes, the mixture is reciprocal and not NRBI. In other words, the tracelessness says that if there is nonreciprocity in the material, its magnitude has to be of different sign for different field directions.

III. The Case of Chromium Oxide

It seems, however, that critical objections can be raised against the traceless property of the nonreciprocity dyadic. First, let us consider the case of chromium oxide (Cr_2O_3) that is known to display the magnetoelectric effect [5], i.e., the electric field induces magnetic polarization in the material (effect labeled as ME_E), and the magnetic field produces electric polarization (ME_H).

In measurements, the magnetoelectric effect in chromium oxide has been shown to be uniaxially anisotropic, and hence one might not expect it to be an example of isotropic nonreciprocal material. However, the effect is also very sensitive to the temperature. In fact, the temperature dependence of the magnetoelectric components along the crystallographic c axis $\alpha_{||}$, and perpendicular to it, $\alpha_{\perp}$, have been measured. Fig. 1 is a reproduction of the measurements[4] made by Folen *et al.* in 1961 [6]. The figure shows that for a large temperature range, the transverse component is much smaller in amplitude than the axial one[5], but since the temperature dependences are opposite to each other, one can find a point (around 120 K) where the two components are equal.[6]

This point corresponds to NRBI behavior in chromium oxide, in contradiction with the claim of [1] which requires that the two components be of opposite sign. One might suggest that there is a sign error in the measurements of [6]. This being the case, the measurement results still are sufficient to prove that the traceless condition does not hold for chromium oxide, because then the two curves should be mirror images of each other (in other words, the amplitude of the longitudinal one being minus two times the transversal one).

One should be aware of the fact that here the trace of $\bar{\bar{\chi}}$ has been analyzed, whereas the results of [1] where derived using the $\bar{E}, \bar{B}$ basis, i.e., the Boys–Post relations. Therefore, strictly taken we should look at the trace of the dyadic $\bar{\bar{\chi}} \cdot \bar{\bar{\mu}}^{-1}$ instead of $\bar{\bar{\chi}}$. This is not possible since the authors of [6] did not report the permeability components. However, because the permeability is very close to the free space value (see footnote 6), the difference between these two dyadics is negligible.

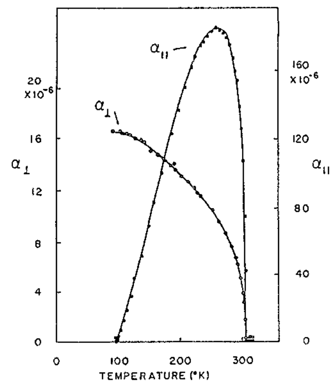

Fig. 1. The temperature dependence curves of the magnetoelectric coefficients of chromium oxide. Measurements by Folen *et al.* in 1961 [6].

To conclude the argument of the possible existence of NRBI behavior of Cr_2O_3, one needs to be sure that the magnetoelectric effect is indeed the nonreciprocal effect. Folen *et al.* only measured the electrically induced magnetoelectric effect. But as is seen from the relations (3)–(4), there are two physical mechanisms responsible for magnetoelectric polarization: the nonreciprocal effect and the chirality effect. The magnetoelectric effect in chromium oxide is nonreciprocal because the effect is an effect measurable for static fields, whereas the chiral effect vanishes in statics [3]. As a matter of fact, chromium oxide does not exhibit natural optical activity [8]. Further support for the purely nonreciprocal character of the magnetoelectric effect is brought by the measurements of O'Dell [9], where he measured both the magnetoelectric components ME_E and ME_H. As can be seen from (3)–(4) for symmetric dyadics (e.g., the present case of uniaxial anisotropy), the two magnetoelectric coefficients are equal in magnitude and sign for nonreciprocal nonchiral materials, and equal and opposite in sign for reciprocal chiral media. O'Dell's measurements [9] yield the following results for these two parameters $-2.20 \cdot 10^{-4}$ ($\pm 10\%$) and $-1.85 \cdot 10^{-4}$ ($+20\%, -10\%$), which constitute conclusive evidence for the nonreciprocal (rather than optically active) character of the magnetoelectric effect of chromium oxide. See also [10] for discussion of the frequency dependence of the magnetoelectric phenomena.

IV. Phenomenological Tellegen Material

Another example of NRBI materials is the phenomenological medium, often called as Tellegen material [11], depicted in Fig. 2. There, the basic element is a doublet of electric and magnetic dipoles, tied parallel together [corresponding to a positive value for the nonreciprocity parameter χ in the relations (3)–(4)], or antiparallel (negative nonreciprocity parameter). Similar elements are randomly dispersed with different orientations in neutral background material, whence the medium is isotropic. The magnetoelectric effect comes from the fact that external electric field exerts a torque on the elements through interaction with the electric moment and at the same time produces magnetic polarization since the magnetic dipole moment also turns (because it is glued to the electric moment). Similarly, an incident magnetic fields brings forth electric polarization.

[3] If materials are treated where the permeability differs considerably from the vacuum value, the nonreciprocity analysis of the present paper can be rewritten easily. The transformation of the magnetoelectric dyadics includes a multiplication with the permeability dyadic.

[4] One of the reasons for using here the constitutive bianisotropic relations (3)–(4) rather than the Boys–Post ones is consistency with the experimental reports: the measurements of Folen *et al.* have been performed by exposing the sample to an electric field and measuring the components of the $\bar{B}$ field rather than the $\bar{H}$ field.

[5] Note the difference in the scales of the figure.

[6] Chromium oxide is antiferromagnetic below the Néel temperature 307 K. However, the analysis can be kept nonmagnetic ($\mu = \mu_0$) for simplicity, since the magnetic susceptibility of this material is around 10^{-3} [7] which means that the permeability is practically equal to the vacuum value.

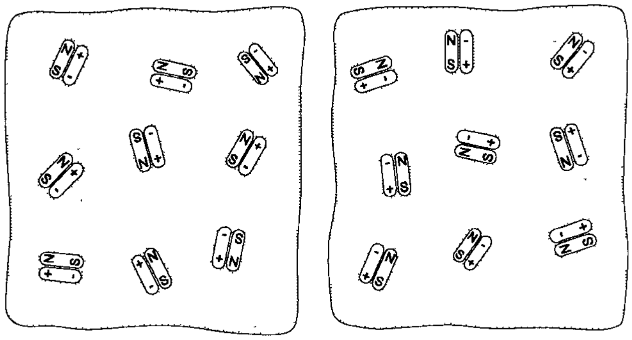

Fig. 2. A phenomenological model for the NRBI Tellegen material. The two samples shown here are isotropic, and have the same magnitude of the nonreciprocity parameter χ but of opposite sign.

This medium evidently is NRBI and violates the result of [1]. There should be no reason why this material could not be built by pivoting the elements on fixed positions within the matrix. The objection can be raised that the interactions between the elements would destroy the isotropy since the symmetry would be spontaneously broken like in permanent magnets. A countermeasure to this danger can be devised by mechanical strings that balance the elements in their original orientations, but which would still allow these to rotate slightly with respect to all three directions in reaction with the incident fields.

Irrespective to the problem of interaction between the elements, one can analyze a single electric-magnetic-dipole pair. This element clearly violates the traceless condition of [1] because its nonreciprocity susceptibility dyadic is uniaxial, being positive for transverse fields and zero for fields along the pair axis. The trace is therefore positive and nonzero.

It is conspicious in this model of Tellegen material that we have to resort to nonelectrical and nonmagnetic forces. This may have some connection to the emphasis in [1] on "physically and chemically stable media" for which the covariance analysis applies. Speculations about unstability of the Tellegen medium also appear in [12], where the asymmetry with respect to time reversal is given as the reason for this.[7]

V. Conclusion

The present article is an attempt to defend the position that NRBI media are not forbidden, at least not because of the covariance principles expounded in [1]. Based on the examples of chromium oxide and phenomenological Tellegen material, the traceless property of the nonreciprocity has been challenged. Since the mathematical derivation leading to the tracelessness in [1] seems to be sound, the arguments of the present paper lead to objections against its starting premise. The inevitable conclusion here is that the covariance property in the form of Post [2] does not hold universally in nature.

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[7]Due to domain nucleation, also simpler media like permanent magnets will eventually decay into the time-symmetric state, thus being unstable. The time constants involved in the process, however, may be millions of years.

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Fourier-Transform Analysis for Rectangular Groove Guide

Byung-Tak Lee, Jae W. Lee, Hyo J. Eom, and Sang-Yung Shin

Abstract—**The rectangular groove guide is analyzed using the Fourier transform and the mode-matching technique. The enforcement of the boundary conditions at the groove apertures yields the simultaneous equations for the field coefficient inside the grooves. The simultaneous equations are solved to represent a dispersion relation in analytic series form. The numerical computation is performed to illustrate the behavior of the guided wave in terms of frequency and groove sizes. The presented series solution is exact and rapidly-convergent so that it is efficient for numerical computation. A simple dispersion relation based on the dominant-mode analysis is obtained and is shown to be very accurate for most practical applications.**

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